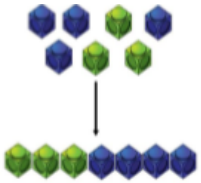
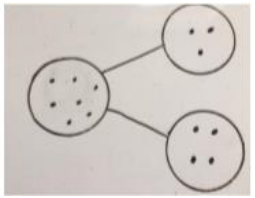
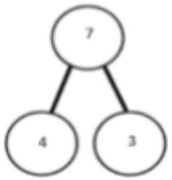
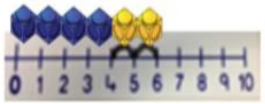
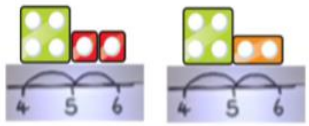
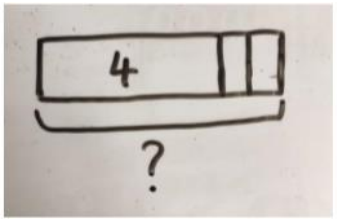
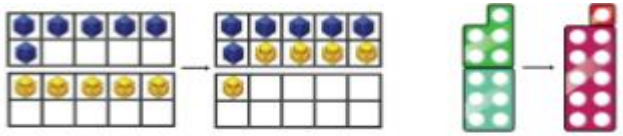
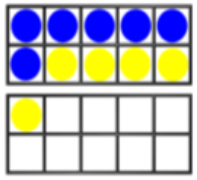
Rationale

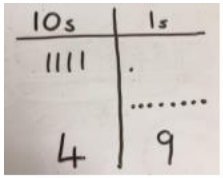
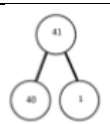
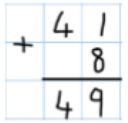
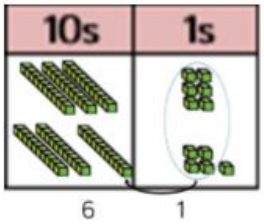
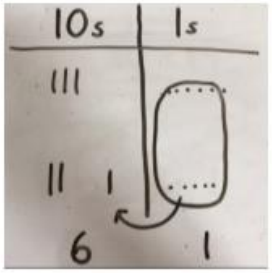
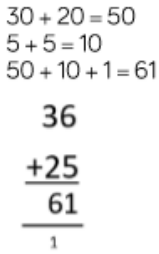
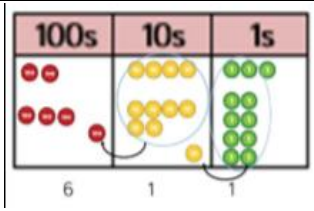
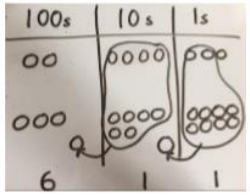
This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the National Curriculum. Through the policy, we aim to link key manipulatives and representations through concreate, pictorial and abstract methods.

School wide policies, such as this, ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching. By providing children with the opportunity to work on different representations of the same mathematical idea allows for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with policy's 'end of year' targets.

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. At the end of each strand, outline the mental strategies that children are expected to develop throughout the school.

Addition

Skills	Concrete	Pictorial	Abstract
Combining two parts to make a whole. **Add on, more than, bigger, most, increase, part, whole, altogether**		 Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.	 4+3=7 Four is a part, 3 is a part and the whole is seven.
Counting on using number lines. **Count on, number line, Numicon, bar model**	 Counting on using number lines using cubes or Numicon. 	 A bar model which encourages the children to count on, rather than count all.	 The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2
Regrouping to make a given number. (Starting with 10). **Bonds, counters**	 Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5.	 Children to draw the ten frame and counters/cubes.	Children to develop an understanding of equality $6 + \square = 11$ $6 + 5 = 5 + \square$ **End of FS target** $6 + 5 = \square + 4$

Two digit add a one digit. (Partition and place value). **Partition, place value**	  Continue to develop understanding of partitioning and place value. 41 + 8		Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.	  *End of Y1 target*
Two digit add a two digit. **Addition, plus**	 Continue to develop understanding of partitioning and place value. 36 + 25.		Chidlren to represent the base 10 in a place value chart.	 Formal method:  Looking for ways to make 10. *End of Y2 target*
Adding a three digit to a three digit number. Use of place value counters. **Exchange**	 When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.		Chidren to represent the counters in a place value chart, circling when they make an exchange.	243 +368 611 1 1 *End of Y3 target* **N.B. See progression in written methods**

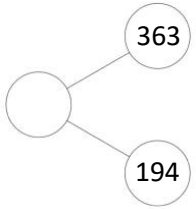
Conceptual variation

363 + 194

☐ = 194 + 363

Three hundred and sixty three plus one hundred and ninety four = ?

?	
194	363



A school owns 363 chairs. They are then given another 194 by the council. How many chairs do they now have?

True/false?

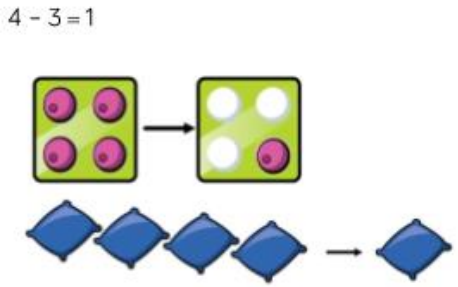
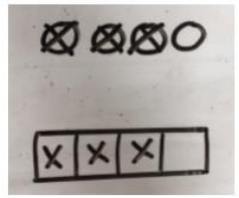
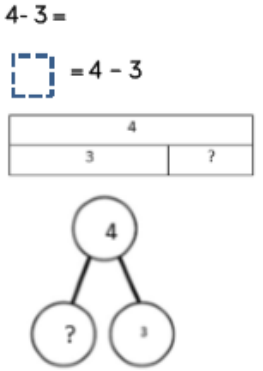
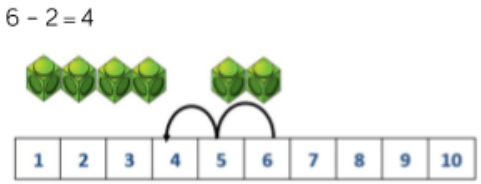
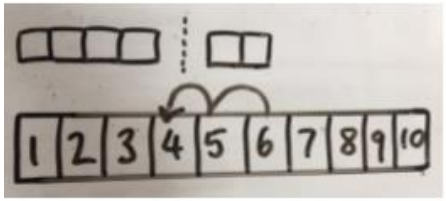
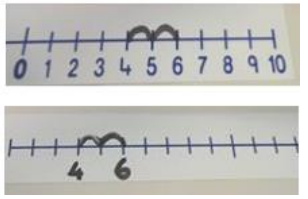
The sum of 363 and 194 = 569.

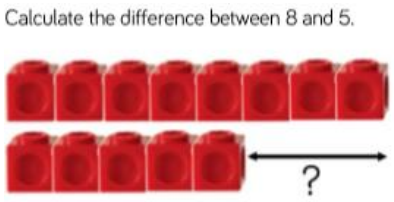
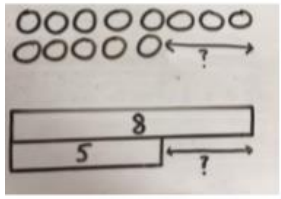
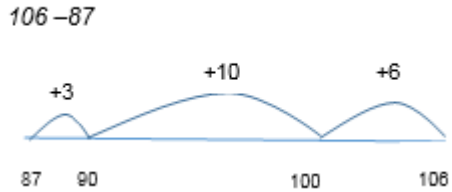
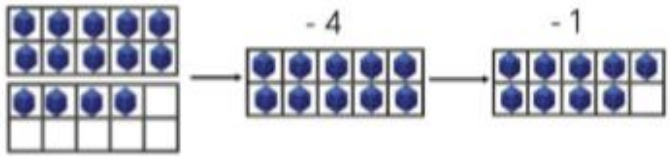
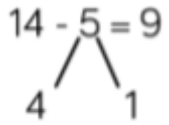
Mental strategies

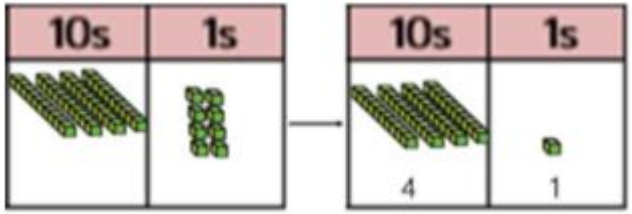
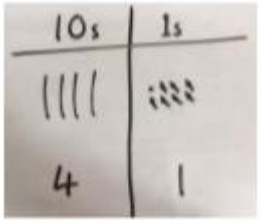
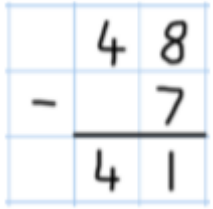
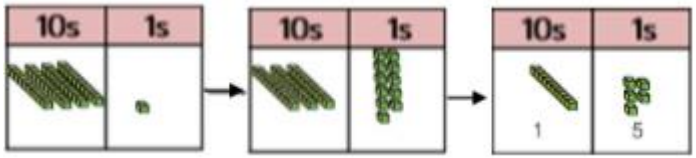
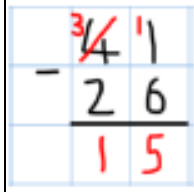
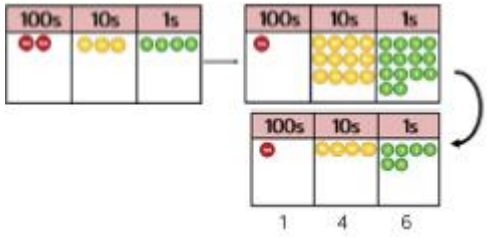
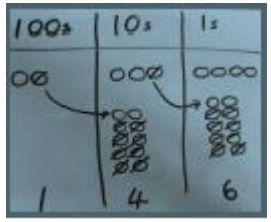
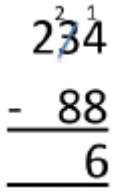
- Count in thousands, hundreds, tens, ones and hundredths as appropraite
- Reorder numbers in a calculation

- Partition into hundreds, tens and ones and in different ways and recombine by breaking units of 6, 7, 8 or 9 into '5 and abit' (724 = 600 +110 +14)
- Add three 1 digit numbers; put the largest number first, using known facts (pairs to 10, doubles)
- Look for near doubles
- Begin to bridge through 10 when, then adjust
- Use known facts and place value to add
- Add 9, 19 and 11 or 21 by rounding and compensating
- Continue to use the relationship between addition and subtraction (encourgaing children to use a bar model)

Subtraction

Skills	Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole **Take away, left, less than, smaller, least, decrease, fewer**	ten frames, Numicon, cubes and other items such as beanbags could be used 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model could also be used. 	
Counting back **Count back, number line** ***Complete <u>subtraction</u> jumps underneath the number line**	Using number lines or number tracks 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track to show their jumps. Encourgae children to use an empty number line. 

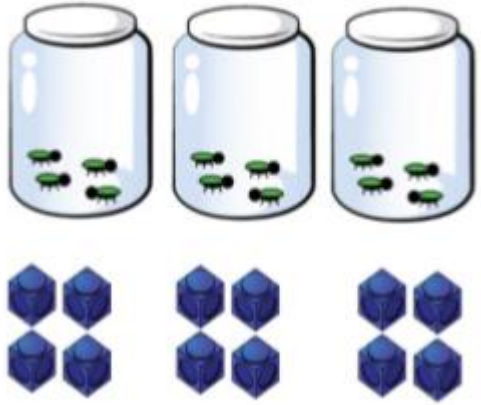
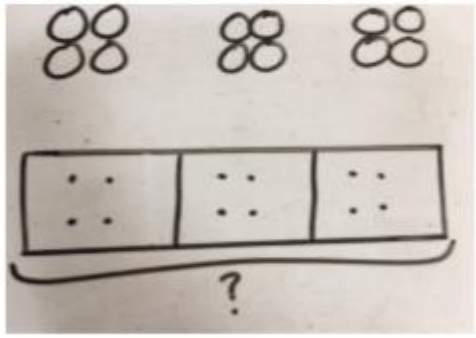
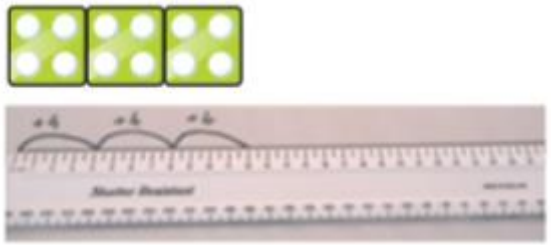
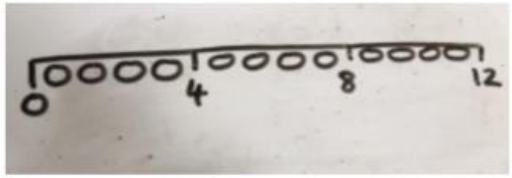
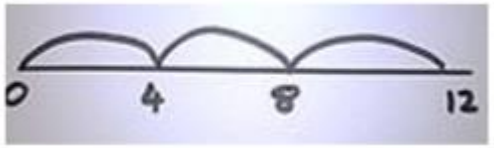
Finding the difference **Difference between, count on**	Using cubes, Numicon or Cuisenaire rods, other objects can also be used 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.  Using counting on as an informal written strategy for subtraction. 	Find the diffence between and 8 and 5. 8 – 5, the difference ☐ Children to explore why 9 – 6 = 8 – 5 = 7 -4 have the same difference. *End of FS target*
Making 10 **Bonds, partition**	Using ten frames 14 -5 	Children to present the ten frame pictorially and discuss whaat they did to make 10. 	Children to show how they can make 10 by partioning the subtrahend.  14 – 4 = 10 10 – 1 = 9 *End of Y1 target*

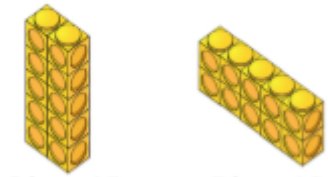
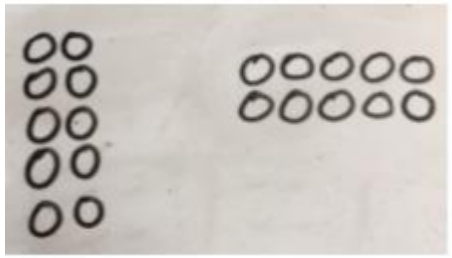
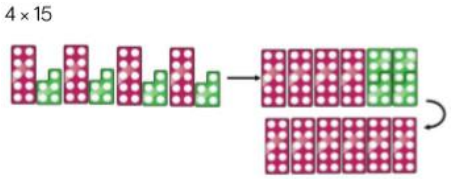
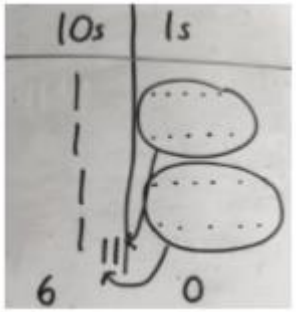
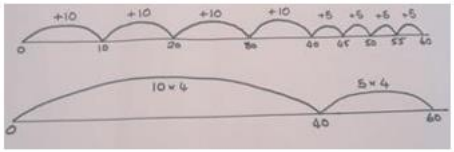
Column method **Subtraction, minus,**	Using base 10 48 -7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7  *End of Y2 target*
Column method **Exchange, column subtraction**	Using base 10 and having to exchange 41-26 	Represent the base 10 pictorially, remembering to show the exchange. 	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41=30+11. 
Column method	Using place value counter 234 – 88 	Representing the place value counters pictorially; remembering what has been exchanged. 	Formal column mehtod. Children must understand what has happened when they have crossed out digits.  *End of Y3 target* *Crossing the thousands barriers End of Y4*

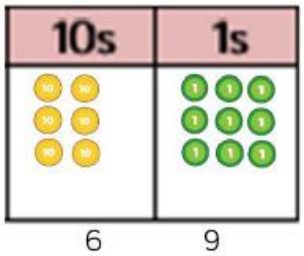
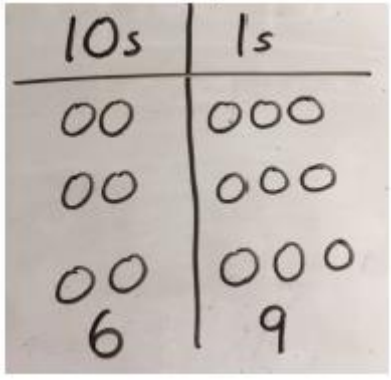
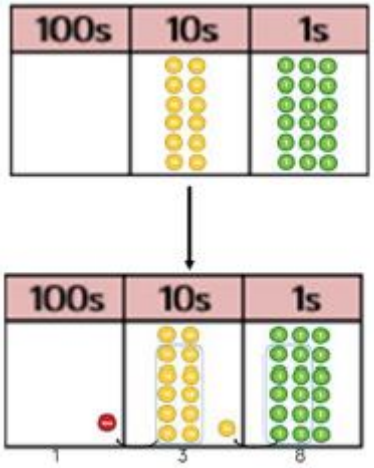
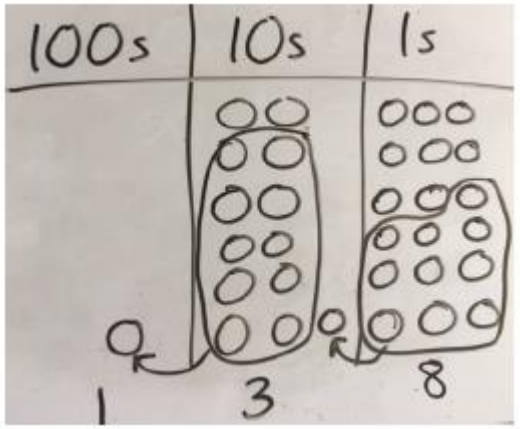
Mental Strategies

- Count back in hundreds, tens or ones
- 1 less than a number, 10 less than a number
- Subtract mentally a near multiple of 10
- Find a small number by counting back
- Find a difference by counting up from the smaller number to the larger number (on a number line)
- Bridge through a multiple of 10 and then adjust
- Use knowledge of number facts and place value to subtract pairs of numbers
- Subtract a 2-digit number by partitioning it and then subtracting its tens and ones
- Use the relationship between addition and subtraction

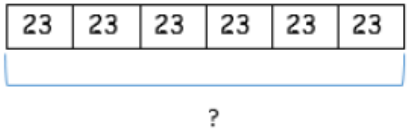
Multiplication

Skills	Concrete	Pictorial	Abstract
Repeated grouping/ repeated addition **grouping, equal groups, group, part, equal, repeated addition How many times?**	3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 x 4 = 12 4 + 4 + 4 = 12 **End of FS target**
Number lines to show repeated groups **groups, groups of, lots of, multiply, multiplied by, times, steps, equal**	3 x 4  Cuisenaire rods could also be used.	Abstract number line representing the three groups of four. 	Abstract number line showing three jumps of four.  3 x 4 = 12 **End of Y1 target**

Use arrays to illustrate commutativity **array, lots of, groups of, commutative, repeated addition, row, column**	Counters, objects, Numicon pegs and other objects can be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent arrays pictorially. 	Children to be able to use an array to write a range of calculations, e.g: $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$ **End of Y2 target**
Partition to multiply **partition, tens, ones, value, groups of, lots of, multiply, multiplied by, times, derive product, scale up commutativity associativity**	Partition to multiply using Numicon, Base 10 or Cuisenaire rods. 	Children to represent the manipulatives pictorially. 	4 × 15 10 5 10 × 4 = 40 5 × 4 = 20 40 + 20 = 60 Children should be encouraged to show their process:  A number line might be used alongside.

Formal written method **partition, tens, ones, place value**	Using place value counters (Base 10 could also be used). 3 x 23 	Children represent the place value counters pictorially. 	Children record their process to show their understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69 *End of Y3 target*
Formal written method	Using place value counters (Base 10 could also be used). 6 x 23 	Children to represent the counters/ Base 10 pictorially. 	6 x 23 = 23 × 6 138 1 1 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1 Answer: 3224 *See progression of skills*

<u>Conceptual Variation:</u> Different ways to ask children to solve 6 x 23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

What is the calculation?
What is the product?

Find the product of 6 and 23

6 × 23 =

□ = 6 × 23

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$$

<u>Mental Strategies</u>

- Counting in multiples
- Repeated addition
- Arrays
- Links to doubling, including doubles to link x2, x4 and x8 tables
- Reorder calculation (**commutative**)
- Using known facts and place value
- Use the rule of **associativity**
- Scaling up using known facts
- Using the relationship betwenn multiplication and division
- Use partitioning and **Distributive Law** to multiply
- Use **factor pairs** and the **Associative Law** to multiply
- Recognise and use square and cube numbers

Which tables to know when?

End of FS *- Double and halving*

End of Y1 – *2, 5 and 10 - to have an understanding the multiplication is repeated addition*

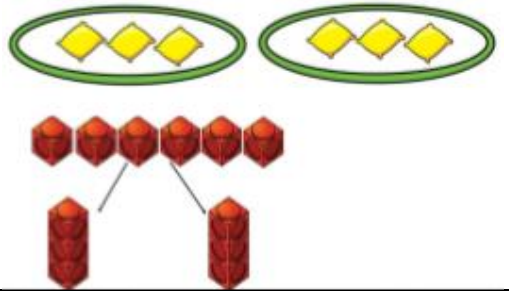
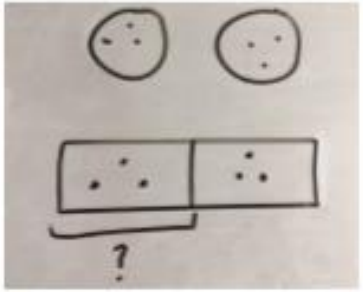
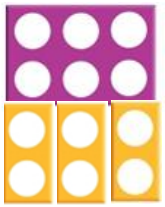
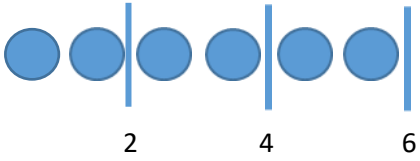
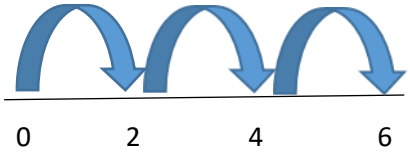
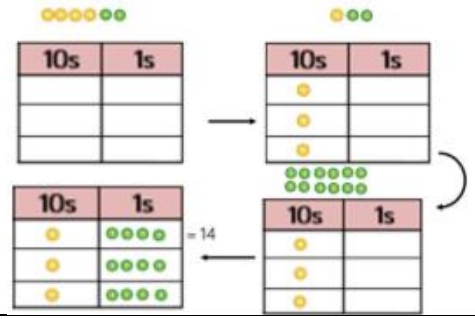
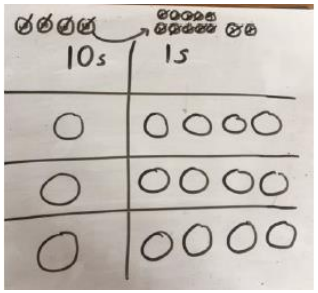
End of Y2 – *3, 4 and 8*

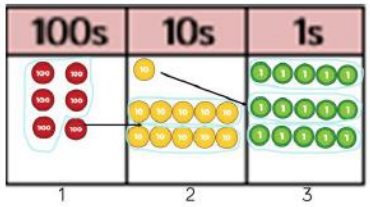
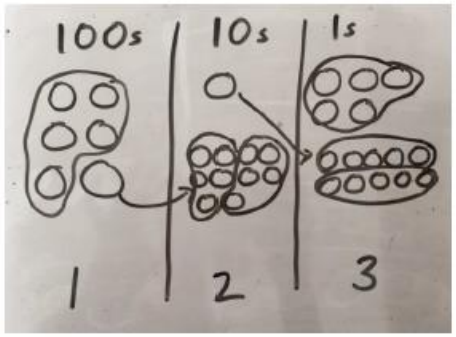
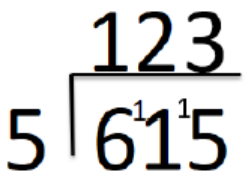
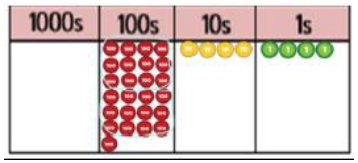
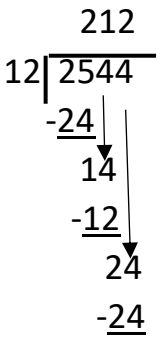
End of Y3 - *6, 9, 7 11*

End of Y4 –- *To recall all facts up to 12 x 12*

****Different to that of symphony due to times table test in year 4****

Division

Skills	Concrete	Pictorial	Abstract
Sharing into equal groups **divide, divided by, share, each, equally, group, groups of, lots of,**	6 ÷ 2 		 Children should be encouraged to also use their 2 x tables facts. 2 x 3 = 6 *End of FS target*
Repeated subtraction **grouping, number line, left, left over, subtract, array**	6 ÷ 2 	Children are encouraged to draw the dots and subtract 2 each time. 	Use a number line to form groups of 2. Times tables should also be applied. *End of Y1 target* 
Sharing using place value counters **place value, counters, exchange, remain, remainder, multiple,**	42 ÷ 3 	Convert from physical manipulation of place value counters to drawing in the table 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14 *End of Y2 target*

Short division **inverse, divisible by, carry, short division, factor, how many groups of __ in __, remainder as fraction prime numbers, factors**	615 ÷ 5  How many groups of 5 hundreds can you make with 6 hundreds? Exchange 1 hundred into 10 tens. How many groups of 5 tens can you make with 11 tens? Exchange the 1 ten for 10 ones. How many groups of 5 ones can you make with 15 ones?	Represent the counters pictorally.  	Children can now use the short division method and carry remainders numerically to complete the calculation.  *End of Y4 target*
Long division **long division, common factor, remainder as decimal, rounded**	2544 ÷ 12   We can't sort two thousands into twelve groups, so we exchange them into thousands. 2 groups of 12 hundred makes 24 hundred. Once subtracted, 1 hundred remains and forms 14 tens. One group of 12 tens can be made, leaving 2 tens to from 24 ones, which makes 2 groups.	 	Children apply their learning of short division, and write the groups underneath to use column subtraction to calculate a remainder. The next digit then meets the remainder rather than carrying the remainder over. Or the double bus stop method. *End of Y6 target*

Mental Strategies

- Count using times tables
- Make links with halving and quartering; use scaling for larger numbers
- Use arrays
- Use known times tables facts and place value
- Use related facts
- Use relationship between x and ÷
- Partition in different ways to divide
- Use factors pairs to simplify original division sum
- Use distributive law to divide (98 ÷7 = ((70÷7) + (28 ÷ 7)
- Counting in steps of powers of 10

Which division facts to know when?

End of FS *- Double and halving*

End of Y1 – *2, 5 and 10 - to have an understanding the multiplication is repeated addition*

End of Y2 – *3, 4 and 8*

End of Y3 - *6, 9, 7 11*

End of Y4 —- *To recall all facts up to 12 x 12*

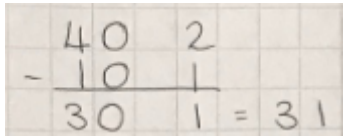
****Different to that of symphony due to times table test in year 4****

Progression of Written Methods for Subtraction

Expanded – no exchange

42 - 11

End of Y2 target



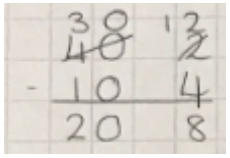
Teaching Point

Encourage children to start at the 'ones' column to feed into later practice.

Expanded – one exchange

42 - 14

End of Y3 target



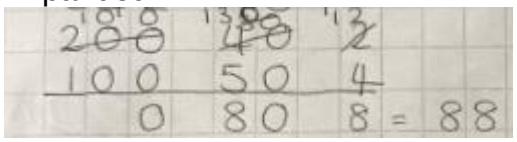
Teaching Point

You are exchanging 1 ten for 10 ones – make this vocabulary explicit.

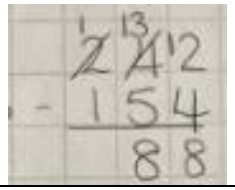
Two exchanges

242 – 154

Expanded:



Compact:



Teaching Point

You can progress to compact method at any step once the child is secure with their place value.

Include a 0

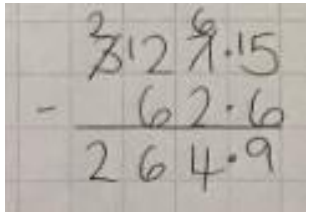
End of Y4 target

Teaching Point

Always include 4-digit – 3-digit to ensure secure place value.

Decimals

327.5 – 62.6

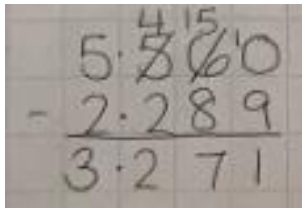


Teaching Point

Children must include the decimal point in the sum, and line them up like buttons on a shirt to ensure correct place value

Decimals to 3d.p in context

5.560 – 2.289



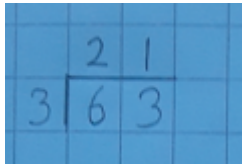
Teaching Point

Encourage use of 0 as a place holder.

Progression of Written Methods for Division

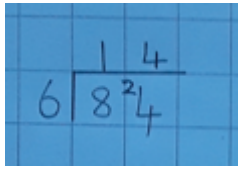
Short division

63 ÷ 3

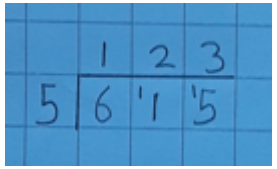


Short division carrying remainders

84 ÷ 6



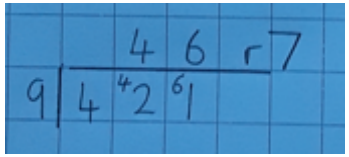
615÷ 5



Short division with remainders

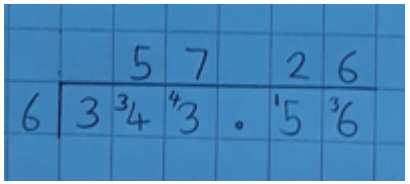
421 ÷ 9

End of Y4 target



Short division with decimal points

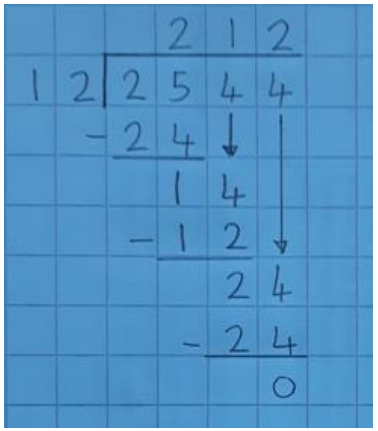
343.56 ÷ 6



Long division

2544 ÷ 12

End of Y6 target



Different methods to approach long division

Factor pairs (Double bus stop)

165 ÷ 15

Teaching Point

Children apply their learning of short division, and write the groups underneath to use column subtraction to calculate a remainder. The next digit then meets the remainder rather than carrying the remainder over.

For decimal long division, add the decimal point before solving the calculation.

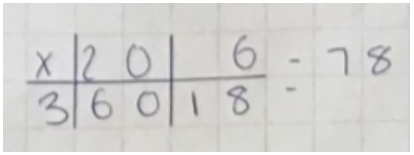
Teaching point

Children record the factor pairs of the 2-digit divisor to support the use of short division as shown above.

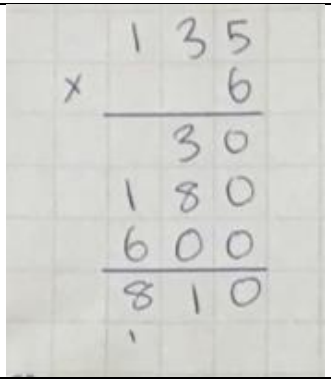
Progression of Written Methods for Multiplication

Grid Method
20 x 3

End of Y3 target



Expanded form
135 x 6

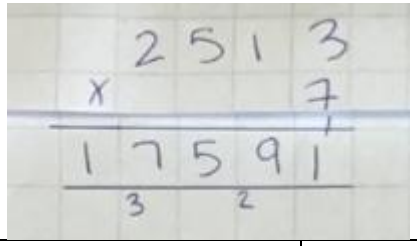


Teaching Point

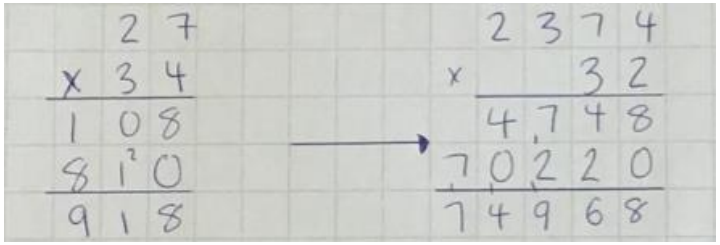
Do not stay here. Quickly progress onto compact as soon as possible

4 digit multiplied by a 1 digit

End of Y4 target



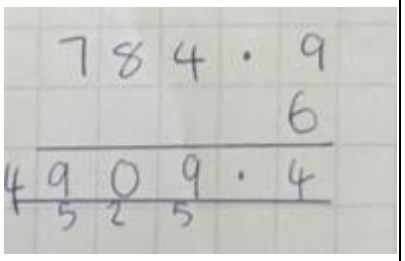
Long multiplication **End of Y5 target**

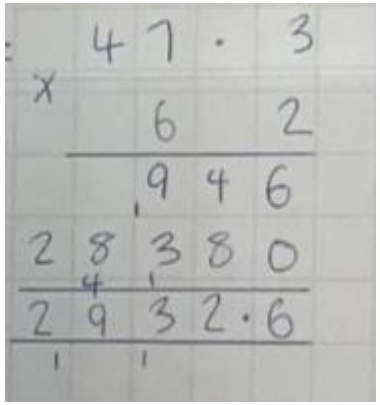


Teaching Point

Make Place value explicit. You are multiplying by 30 not 3. Multiples of 10 end in a zero, we can add the zero at the start.

Multiplying decimals by integers – apply context of money and measure





Teaching Point

Ignore DP to start with. Complete calculation and then count DP back in e.g. 1DP in questions means = 1DP in answer.

Teaching Point

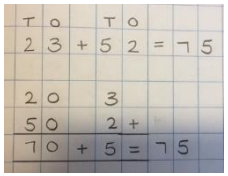
Ignore DP. Make it clear to line up as normal 3 digit x 2 digit, then count in DP after, otherwise place value is confused and method can become compromised.

Progression in Written Methods for Addition

Partitioning 2d+2d

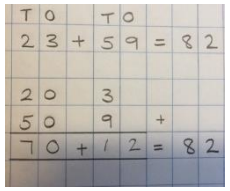
Children need a secure understanding of place value.

23+52 (no exchange)



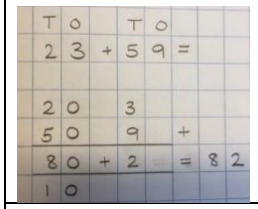
Teaching Point
Start with adding ones. Reinforce place value columns language.

23+59 (1s crossing 10)



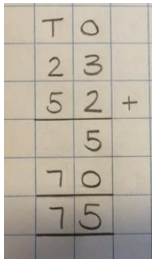
End of Y2 target

23+59 (exchange 1s)



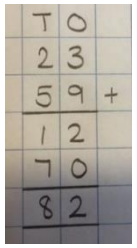
Expanded 2d+2d

23+52 (no exchange)



Teaching Point
Start with adding ones.
3+2=5
20+50=70
Reinforce recording in 1s and 10s place.

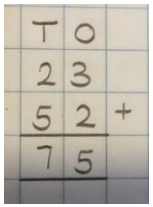
23+59 (1s crossing 10)



Teaching Point
The total of 3 and 9 is 12. The 1 digit in 12 is in the 10s place so the 1 digit is recorded in the 10s column (place).

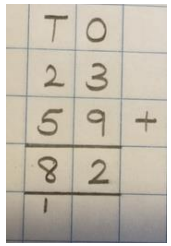
Compact 2d+2d

23+52 (no exchange)



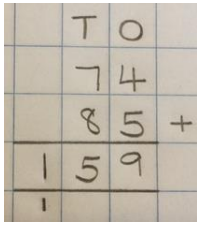
Teaching Point
After adding 1s reinforce that the 2 and 5 digits are in the 10s column so the total of 7 in the tens columns has a value of 70.

23+59 (exchange 1s)



Teaching Point
Total of ones is 12 so we exchange 10 ones for 1 ten and this is shown by placing the 1 digit in the tens column (as shown).

74+85 (exchange 10s)



Teaching Point
7 tens + 8 tens = 15 tens. Exchange 10 tens for 1 hundred which is recorded in the hundreds column.

46+89 (exchange 1s and 10s)

Ensure children progress from no exchange then 1 exchange, 2 exchanges, 3 exchanges, various exchanges.

- Add with up to 3 digit numbers using formal written method. **End of Y3 target**
- Add with up to 4 digit numbers using formal written method. **End of Y4 target**
- Add with more than 4 digit numbers using formal written method. **End of Y5 target**

Introduce addition with decimal points using the context of money or measures.

Also include:

- Add more than two numbers
- Mixed number of digits (e.g. 3d+4d+3d)

Important to Consider

Model 'thinking out loud' about which strategy to use so that children can start to reflect on their choice of strategies. *Can I do this calculation in my head using a mental strategy? What is the most efficient strategy? Can I use jottings? Do I need to use a formal written method?*